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RECYCLING AND EXHAUSTIBLE RESOURCES

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1. Introduction

A previous paper [2] dealt with the economics of extracting an exhaustible resource when there exists a high-cost inexhaustible substitute technology which must ultimately become economical to operate in conjunction with primary extraction. Solar energy as an alternative to the recovery of fossil fuels and the recycling of material resources such as iron or copper as an alternative to mining represent examples under these assumptions. However, in the case of recycling the model did not allow for the possibility that recycled output, and therefore cost, may depend upon the floating stock of recyclable material. This turns out to be a particularly interesting and appropriate case, whose development will be the objective of this note. For additional analyses of the problem of recycling the reader is referred to Smith [3] and D'Arge and Kogiku [1].

2. Optimal Mining and Scrap Recycling

Assume that a variable nonproducible resource such as labor is available in fixed total quantity $\bar{L}$. This resource must be continuously allocated between (1) mining and refining primary ore stock, and (2) collecting and recycling the scrap residue of products made from previously mined material. Let $L(t)$ be the allocation of this resource

at time t to mining, and $\bar{L} - L(t)$ be the allocation to recycling activity. If the stock of unrecovered primary ore, $Q(t)$, and the stock of scrap are measured in the same units (e.g. tons of refined steel), and $\bar{Q}$ is the initial stock of the earth deposit, then $\bar{Q} - Q(t)$ is the floating stock of scrap available for recycling. The specification that the total available stock of refinable material must always be present either in the form of primary ore or scrap follows from (1) the law of the conservation of mass under ordinary thermal, chemical, and mechanical transformations; and (2) the assumption that the commodity produced from refined material has zero durability, i.e. it is consumed as an instantaneous flow.

In effect society must choose between mining primary earth deposits, and "mining" secondary surface stocks of scrap. In each case we assume that the commodity output rate is an increasing concave function of the labor employed and the material stock being mined or recycled. We also assume that each technology exhibits constant returns to scale. Consequently, the output of commodity, q_1, from recycling can be written

$$q_1 = (\bar{Q} - Q)\Psi\left(\frac{\bar{L} - L}{\bar{Q} - Q}\right), \quad \Psi' > 0, \quad \Psi'' < 0.$$

while the output, q_2, from earth ores is

$$q_2 = Q\phi(L/Q), \quad \phi' > 0, \quad \phi'' < 0.$$

It should be noted that while the law of the conservation of mass guarantees that all previously mined raw material remains available for recycling, the law of diminishing returns assures us that increasing increments of effort are required to yield additional increments in the output rate from any given stock of scrap.

Since the stock of earth ore is finite and depletes at the rate q_2, we must have

$$\dot{Q} = -Q\phi(L/Q).$$

For any given level of mining effort, L, the stock of ore will decline, while the stock of scrap must increase at the same rate. If instantaneous concave social utility is $u(q_1 + q_2)$, then the problem for society can be posed as one of maximizing the discounted utility,

$$\int_0^\infty u(q_1 + q_2)e^{-\delta t}dt$$

subject to $\dot{Q} = -Q\phi(L/Q),\ L \geq 0,\ Q \geq 0.$

The Hamiltonian is now

$$H = u[(\bar{Q} - Q)\Psi(\frac{\bar{L} - L}{\bar{Q} - Q}) + Q\phi(L/Q)] + \xi[-Q\phi(L/Q)]$$

where L is the control variable, Q the state variable, and ξ the auxiliary state variable representing the value of a unit of ore reserves net of its opportunity cost as recyclable scrap after recovery. The fact that ore stocks, once mined and refined, become part of the stock of recyclable scrap serves to reduce the "conservation" value of the exhaustible resource. The price of scrap is thus an opportunity cost of conserving a unit of the resource.[1] In what follows, the derivations will be much simplified by assuming $u' = 1$, so that the criterion is to maximize the initial present value of output.

An optimal trajectory [3] must satisfy the following:[2]

$$\text{If } \frac{\partial H}{\partial L} = (1 - \xi)\phi' - \Psi' \begin{cases} > 0,\ L = \bar{L} & \text{(1a)} \\ = 0,\ 0 \leq L \leq \bar{L} & \text{(1b)} \\ < 0,\ L = 0. & \text{(1c)} \end{cases}$$

$$\dot{\xi} = \delta\xi - \frac{\partial H}{\partial Q} = \delta\xi - (1 - \xi)[\phi - (L/Q)\phi'] + [\Psi - (\frac{\bar{L} - L}{\bar{Q} - Q})\Psi'] \quad (2)$$

$$\dot{Q} = -Q\phi(L/Q) \quad (3)$$

$$\lim_{t\to\infty} e^{-\delta t}\xi(t) \geqq 0,\quad \lim_{t\to\infty} e^{-\delta t}\xi(t)Q(t) = 0 \quad (4)$$

Setting $L = \bar{L}$ in (1b) defines the function

$$\xi = b_2(Q) \equiv 1 - \Psi_0' / \phi'(\bar{L}/Q),$$

which is the separating boundary between region III (Figure 1), defining the set of states in which the economy specializes in mining (no scrap recycling) and region II in which the economy carries out mining and recycling activity simultaneously. When L = 0 in (1b), we have

$$\xi = b_1(Q) = 1 - \Psi'(\frac{\bar{L}}{\bar{Q} - Q}) / \phi_0', \quad (5)$$

separating region II from I, in which the mining of earth ores has been displaced by recycling. Assuming $\phi_0' > \Psi_0'$, the following properties of these functions are shown in the illustration of Figure 1:

$$\lim_{Q\to 0} b_2(Q) = -\infty,\quad \lim_{Q\to\infty} b_2(Q) = 1 - \Psi_0'/\phi_0',\quad b_2'(Q) > 0;$$

$$\lim_{Q\to 0} b_1(Q) = 1 - \Psi'(\bar{L}/\bar{Q})/\phi_0',\quad \lim_{Q\to\bar{Q}} b_1(Q) = 1,\quad b_1'(Q) > 0.$$

From the various solutions specified in (1a) - (1c) the differential equations (2) and (3) can be expressed

$$\dot{\xi} = \begin{cases} \delta\xi + [\Psi - (\frac{\bar{L}}{\bar{Q} - Q})\Psi'],\quad \xi \geq b_1(Q). & \text{(6a)} \\ \delta\xi - (1 - \xi)[\phi - (L/Q)\phi'] + [\Psi - (\frac{\bar{L} - L}{\bar{Q} - Q})\Psi'],\quad b_2(Q) < \xi < b_1(Q). & \text{(6b)} \\ \delta\xi - (1 - \xi)[\phi - (\bar{L}/Q)\phi'],\quad \xi \leq b_2(Q). & \text{(6c)} \end{cases}$$

$$\dot{Q} = \begin{cases} 0, & \xi \geq b_1(Q) \quad (7a) \\ -Q\phi(L/Q), & b_2(Q) < \xi < b_1(Q) \quad (7b) \\ -Q\phi(\bar{L}/Q, & \xi \leq b_2(Q) \quad (7c) \end{cases}$$

From (7) it is seen that the resource stock reaches a stationary equilibrium anywhere on or above $b_1(Q)$, and wherever $Q = 0$. Everywhere else mining is carried out at some positive level, and $\dot{Q} < 0$. Figure 1 illustrates $b_1(Q)$ which represents the stationary state resource stock supply function.

From (6a), $\dot{\xi} > 0$ for all $\xi \geq b_1(Q)$. If a path enters region I at some $Q^{\dagger} > 0$, then

$$\dot{\xi} = \delta\xi + \left[\Psi - \left(\frac{\bar{L}}{\bar{Q} - Q^{\dagger}}\right)\Psi'\right],$$

and ξ rises exponentially thereafter. In region II, from (6b), the set of points $\{(\xi, Q) \mid \dot{\xi} = 0\}$ defines the stationary state demand for the resource stock, $Q = d(\xi)$. Setting $\dot{\xi} = 0$ in 5(b), and differentiating (1b) and (6b), the resource demand function has a negative slope:

$$\frac{dQ}{d\xi} = \frac{(\delta + \phi)[(1 - \xi)(\phi''/Q) + \Psi''/(\bar{Q} - Q)] + \phi'\Psi''\bar{L}/(\bar{Q} - Q)^2}{\frac{(1 - \xi)\phi''\Psi''}{Q(\bar{Q} - Q)}\left[\frac{L}{Q} - \left(\frac{\bar{L} - L}{\bar{Q} - Q}\right)\right]^2} < 0.$$

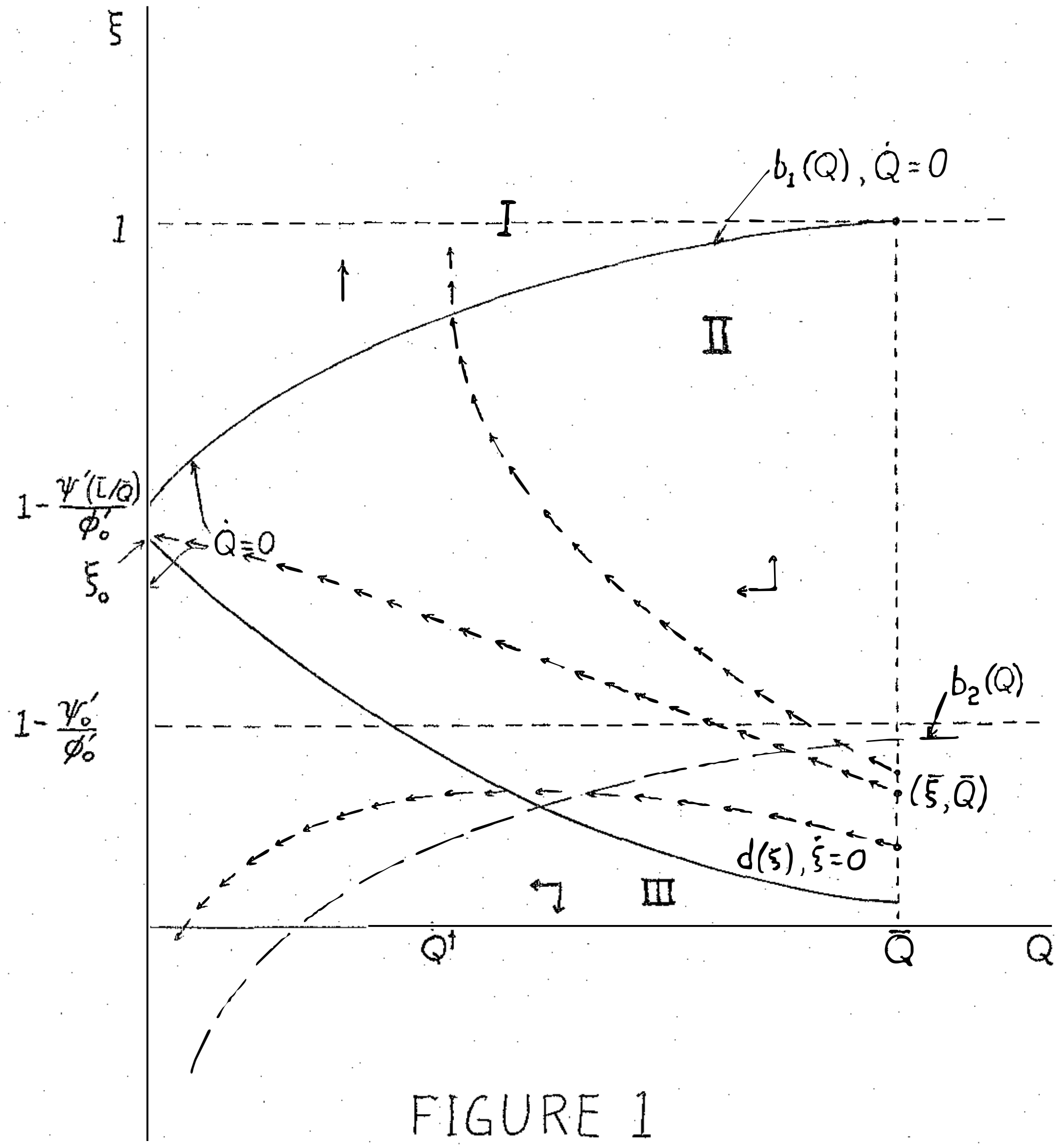
Similarly, for region III, setting $\dot{\xi} = 0$ in (6c) defines the stationary demand for the resource, with $dQ/d\xi < 0$. Figure 1 illustrates the resource demand function $d(\xi)$ in regions II and III.

The functions $b_1(Q)$ and $d^{-1}(Q)$ cannot have a point in common for $Q > 0$. This can be seen by examining (5), which defines $b_1(Q)$, and (1b), which with (6b) when $\dot{\xi} = 0$, defines $d(\xi)$. From (5) and (1b), since Ψ' and ϕ' are monotone decreasing,

$$d^{-1}(Q) \equiv 1 - \Psi'\left(\frac{\bar{L} - L}{\bar{Q} - Q}\right) / \phi'(L/Q) < 1 - \Psi'\left(\frac{\bar{L}}{\bar{Q} - Q}\right) / \phi_0' \equiv b_1(Q),$$

for all $Q > 0$, $L > 0$. Hence $d^{-1}(Q)$ is everywhere below $b_1(Q)$ as shown in Figure 1.

An equilibrium path with initial $Q = \bar{Q}$ must begin at $\bar{\xi}$ such that the trajectory strikes $\xi = \xi_0$, when $Q = 0$. All other paths diverge and are nonoptimal [3].



ξ
1
I
$b_1(Q)$, $\dot{Q}=0$
II
$1-\frac{\psi'(\bar{L}/Q)}{\phi_0'}$
$\dot{Q}=0$
ξ_0
$1-\frac{\psi_0'}{\phi_0'}$
$b_2(Q)$
$(\bar{\xi},\bar{Q})$
$d(\xi)$, $\dot{\xi}=0$
III
$Q^\dagger$
$\bar{Q}$
Q

FIGURE 1

FOOTNOTES

1. This can be seen mathematically by letting $S = \bar{Q} - Q$ be the stock of scrap, and adding the second differential equation $\dot{S} = -\dot{Q} = Q\phi(L/Q)$. The Hamiltonian is then $H = u + pQ\phi(L/Q) - qQ\phi(L/Q)$ where p is the price of the scrap stock, S, and q the price of the ore reserve stock, Q. In the text nothing essential is changed by the fact that we have substituted $S = \bar{Q} - Q$, and defined $\xi = q - p$ as the price of ore reserves minus the price of scrap.

2. The sufficient conditions for a maximum are also satisfied, i.e. for the maximizing value of L, determined by (1), H is concave in Q for given (ξ, t).

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